High School Algebra

High School Algebra Course Description

The fundamental purpose of this course is to formalize and extend the mathematics that students learned in the middle grades. Because it is built on the middle grades standards, this is a more ambitious version of Algebra I than has generally been offered. The critical areas, called units, deepen and extend understanding of linear and exponential relationships by contrasting them with each other and by applying linear models to data that exhibit a linear trend, and students engage in methods for analyzing, solving, and using quadratic functions. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

Inquiry and Application Standards

In response to higher standards and real-world demands, there exists a growing need across content areas and grade levels for students to become resourceful, effective investigators and problem-solvers. Inquiry-based teaching is a powerful vehicle through which such goals for learning are possible (Barron et al., 1998; Edelson, Gordin, & Pea, 1999; Lappan, 2000; Owens, Hester, & Teale, 2002; Perez, 2000).

Inquiry-based learning encourages creative problem solving and risk taking in mathematics (Perez, 2000).

Inquiry-based teaching is an instructional approach in which students' own interests and curiosities drive the learning process and products. Students select topics to research; formulate questions; collect, cull and synthesize information; and, finally, create and present a product that has real-world application (such as models, interviews, experiments) from what they learned. Teachers serve as facilitators and resources (Owens et al., 2002)

Oakwood City School District
High School Algebra

Content Standards

I. Seeing Structure in Expressions

A. Interpret the structure of expressions

1. Interpret expressions that represent a quantity in terms of its context. (a) Interpret parts of an expression, such as terms, factors, and coefficients. (b) Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1 + r)^n$ as the product of P and a factor not depending on P.

2. Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 – y^4$ as $(x^2)^2 – (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.

B. Write expressions in equivalent forms to solve problems

1. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression

   a) Factor a quadratic expression to reveal the zeros of the function it defines

   b) Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

   c) Use the properties of exponents to transform expressions for exponential functions. For example the expression 1.15^t can be written as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%

2. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments.

II. Arithmetic with Polynomials and Rational Expressions

A. Perform arithmetic operations on polynomials

1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

B. Understand the relationship between zeros and factors of polynomials

1. Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x – a is p(a), so p(a) = 0 if and only if (x – a) is a factor of p(x).

2. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

C. Use polynomial identities to solve problems

1. Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.

2. (+) Know and apply the Binomial Theorem for the expansion of (x + y)n in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.

D. Rewrite rational expressions

1. Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.

2. (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

III. Creating Equations

A. Create equations that describe numbers or relationships

1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.

4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law V = IR to highlight resistance R.

IV. Reasoning with Equations and Inequalities

A. Understand solving equations as a process of reasoning and explain the reasoning

1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

B. Solve equations and inequalities in one variable

1. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

2. Solve quadratic equations in one variable.

a) Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x – p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.

b) Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them in $a \pm bi$ for real numbers a and b.

C. Solve systems of equations

1. Prove that a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

2. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

3. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.

4. (+) Represent a system of linear equations as a single matrix equation in a vector variable.

5. (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3x3 or greater)

D. Represent and solve equations and inequalities graphically

1. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

2. Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

3. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.