Oakwood City School District
High School Functions Integrated Mathematics

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Functions describe situations where one quantity determines another. Because we continually make theories about dependencies between quantities in nature and society, functions are important tools in the construction of mathematical models. Functions usually have numerical inputs and outputs and are often defined by an algebraic expression. A function can be described in various ways, such as by a graph, a verbal rule, an algebraic expression, or by a recursive rule.

Inquiry and Application Standards

In response to higher standards and real-world demands, there exists a growing need across content areas and grade levels for students to become resourceful, effective investigators and problem-solvers. Inquiry-based teaching is a powerful vehicle through which such goals for learning are possible (Barron et al., 1998; Edelson, Gordin, & Pea, 1999; Lappan, 2000; Owens, Hester, & Teale, 2002; Perez, 2000).

Inquiry-based learning encourages creative problem solving and risk taking in mathematics (Perez, 2000).

Inquiry-based teaching is an instructional approach in which students' own interests and curiosities drive the learning process and products. Students select topics to research; formulate questions; collect, cull and synthesize information; and, finally, create and present a product that has real-world application (such as models, interviews, experiments) from what they learned. Teachers serve as facilitators and resources (Owens et al., 2002)

Content Standards

I. Interpreting Functions

A. Understand the concept of a function and use function notation

1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, the f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).

2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fiboonacci sequence is defined recursively by F(0) = F(1) = 1, f(n + 1) = f(n) + f (n – 1) for n ≥ 1.

B. Interpret functions that arise in applications in terms of the context

1. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

2. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.

3. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

C. Analyze functions using different representations

1. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

a) Graph linear and quadratic functions and show intercepts, maxima, and minima.

b) Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

c) Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

d) (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

e) Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

2. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

a) Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

b) Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^5$, $y = (0.97)^5$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay.

3. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

II. Building Functions

A. Build a function that models a relationship between two quantities

1. Write a function that describes a relationship between two quantities. (a) Determine an explicit expression, a recursive process, or steps for calculation from a context. (b) Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential and relate these functions to the model. (c) (+) Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.

2. Write arithmetic and geometric sequences both recursively and with an explicit formula; use them to model situations, and translate between the two forms.

B. Build new functions from existing functions

1. Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

2. Find inverse functions.

a) Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, $f(x) = 2x^3$ for x>0 or f(x) = (x+1)/(x-1) for x ≠1.

b) (+) Verify by composition that one function is the inverse of another.

c) (+) Read values of an inverse function from a graph or table, given that the function has an inverse.

d) (+) Produce an invertible function from a non-invertible function by restricting the domain.

3. (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

III. Linear and Exponential Models

A. Construct and compare linear and exponential models and solve problems

1. Distinguish between situations that can be modeled with linear functions and with exponential functions.

a) Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.

b) Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

c) Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table.)

3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

4. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.

B. Interpret expressions for functions in terms of the situation they model

1. Interpret the parameters in a linear or exponential function in terms of a context.

IV. Trigonometric Functions

A. Extend the domain of trigonometric functions using the unit circle

1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$, and $\pi/6$, and use the unit circle to express the values of sine, cosines, and tangent for x, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number.

4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

B. Model periodic phenomena with trigonometric functions

1. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

2. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.

3. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of context.

C. Prove and apply trigonometric identities

1. Prove the Pythagorean identity and use it to calculate trigonometric ratios. $\sin^2(\theta)+\cos^2(\theta)=1$

2. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.