

# Oakwood City School District

## High School Number and Quantity Integrated Mathematics

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Students in High School will be exposed to an extension of number, when the real numbers are augmented by the imaginary numbers to form the complex numbers. With each extension of number, the meanings of addition, subtraction, multiplication, and division are extended. In each new number system – integers, rational numbers, real numbers, and complex numbers – the four operations stay the same in two important ways: They have communicative, associative, and distributive properties and their new meanings are consistent with their previous meanings.

### Inquiry and Application Standards

In response to higher standards and real-world demands, there exists a growing need across content areas and grade levels for students to become resourceful, effective investigators and problem-solvers. Inquiry-based teaching is a powerful vehicle through which such goals for learning are possible (Barron et al., 1998; Edelson, Gordin, & Pea, 1999; Lappan, 2000; Owens, Hester, & Teale, 2002; Perez, 2000).

Inquiry-based learning encourages creative problem solving and risk taking in mathematics (Perez, 2000).

**Inquiry-based teaching** is an instructional approach in which students' own interests and curiosities drive the learning process and products. Students select topics to research; formulate questions; collect, cull and synthesize information; and, finally, create and present a product that has real-world application (such as models, interviews, experiments) from what they learned. Teachers serve as facilitators and resources (Owens et al., 2002)

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### Content Standards

#### I. The Real Number System

##### A. Extend the properties of exponents to rational exponents

1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.
2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.

##### B. Use properties of rational and irrational numbers

1. Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

#### II. Quantities

##### A. Reason quantitatively and use units to solve problems

1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
2. Define appropriate quantities for the purpose of descriptive modeling.
3. Choose a level of accuracy appropriate to limitations on measurement reporting quantities

#### III. The Complex Number System

##### A. Perform arithmetic operations with complex numbers

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1. Know there is a complex number  $i$  such as  $i^2 = -1$ , and every complex number has the form  $a + bi$  with  $a$  and  $b$  real.
2. Use the relation  $i^2 = -1$  and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
3. (+) Find the conjugate of complex number; use conjugates to find moduli and quotients of complex numbers. (This standard is suggested to be considered part of the next cluster).

### B. Represent complex numbers and their operations on the complex plane

1. Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.
2. Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. For example,  $(1 - \sqrt{3}i)^3 = 8$  because  $(1 - \sqrt{3}i)$  has modulus 2 and argument  $120^\circ$ .
3. Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.

### C. Use complex numbers in polynomial identities and equations

1. Solve quadratic equations with real coefficients that have complex solutions
2. (+) Extend polynomial identities to the complex numbers. For example, rewrite  $x^2 + 4$  as  $(x + 2i)(x - 2i)$
3. (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

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### IV. Vector and Matrix Quantities

#### A. Represent and model with vector quantities

1. (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g.,  $v$ ,  $|v|$ ,  $\|v\|$ ,  $v$ ).
2. (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
3. (+) Recommended to move to the next cluster

#### B. Perform operations on vectors

1. (+) Solve problems involving velocity and other quantities that can be represented by vectors (Moved from N-VM cluster 1)
2. (+) Add and subtract vectors.
  - a) Add vectors end – to – end, component wise and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.
  - b) Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.
  - c) Understand vector subtraction  $v - w$  as  $v + (-w)$  is the additive inverse of  $w$ , with the same magnitude as  $w$  and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component - wise.
3. (+) Multiply a vector by a scalar
  - a) Represent scalar multiplication graphically by scaling vectors and possibly reversing their directions; perform scalar multiplication component – wise, e.g., as  $c(v_x, v_y) = (cv_x, cv_y)$ .
  - b) Compute the magnitude of a scalar multiple  $cv$  using  $\|cv\| = |c|v$ . Compute the direction of  $cv$  knowing that when  $|c|v \neq 0$ , the direction of  $cv$  is either along  $v$  (for  $c > 0$ ) or against  $v$  (for  $c < 0$ )

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### C. Perform operations on matrices and use matrices in applications

1. (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.
2. (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.
3. (+) Add, subtract, and multiply matrices of appropriate dimensions.
4. (+) Understand that, unlike multiplications of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.
5. (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
6. (+) Multiply a vector (regarded as a matrix with one column) by matrix suitable dimensions to produce another vector. Work with matrices as transformations of vectors.
7. (+) Work with  $2 \times 2$  matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.